# Warm inflation in the DGP brane-world model

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## Abstract

Warm inflationary universe models on a warped Dvali-Gabadadze-Porrati brane are studied. General conditions required for these models to be realizable are derived and discussed. By using an effective exponential potential we develop models for constant and variable dissipation coefficient ratio $r = \frac{\Gamma}{3H}$. We use recent astronomical observations for constraining the parameters appearing in our models.

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## I. INTRODUCTION

It is well know that warm inflation, as opposed to the conventional cool inflation, presents the attractive feature that it avoids the reheating period [1]. In these kind of models dissipative effects are important during the inflationary period, so that radiation production occurs concurrently together with the inflationary expansion. If the radiation field is in a highly excited state during inflation, and this has a strong damping effect on the inflaton dynamics, then, it is found a strong regimen of warm inflation. Also, the dissipating effect arises from a friction term which describes the processes of the scalar field dissipating into a thermal bath via its interaction with other fields. Warm inflation shows how thermal fluctuations during inflation may play a dominant role in producing the initial fluctuations necessary for Large-Scale Structure (LSS) formation. In these kind of models the density fluctuations arise from thermal rather than quantum fluctuations [2]. These fluctuations have their origin in the hot radiation and influence the inflaton through a friction term in the equation of motion of the inflaton scalar field [3]. Among the most attractive features of these models, warm inflation end when the universe heats up to become radiation domination; at this epoch the universe stops inflating and "smoothly" enters in a radiation dominated Big-Bang phase[1]. The matter components of the universe are created by the decay of either the remaining inflationary field or the dominant radiation field [4].

In the Dvali-Gabadadze-Porrati (DGP) model [5] the induced gravity brane-world was put forward as an alternative to the Randall-Sundrum (RS) one-brane model [6], in which general relativity was recovered, also despite an infinite extra dimension, but without warping in 5-dimensional Minkowski space-time. In the DGP model, the gravitational behaviors on the brane are commanded by the competition between the 5D curvature scalar in the bulk and the 4D curvature scalar on the brane. In contrast to the RS case with high energy modifications to general relativity, the DGP brane produced a low energy modification. In the DGP model, according to the embedding of the brane in the bulk, there appear two branches of background solutions. For a review of the phenomenology of DGP model, see[7] and inflation models and reheating in this scenario were studied in [8, 9, 10, 11].

Usually, in any models of warm inflation, the scalar field, which drives inflation, it is the standard inflaton field. As far as we know, a model in which warm inflation on a warped DGP brane has not been yet studied. The main goal of the present work is to investigate

the possible realization of a warm inflationary universe model, where the energy densities (inflaton-radiation) are confined to the brane in DGP model. In this way, we study warm-DGP model and the cosmological perturbations, which are expressed in term of different parameters appearing in our model. These parameters are constrained from the WMAP three year data [12].

The outline of the paper is a follows. The next section presents a short review in the Friedmann equation on the warped DGP inflation model. In Section III we present the warm inflationary phase on the DGP brane. Section IV deals with the scalar and tensor perturbations, respectively. In Section V we use an exponential potential for obtaining explicit expression for our models. Finally, Sec.VI summarizes our findings. We chose units so that $c = \hbar = 1$.

## II. FRIEDMANN EQUATION ON THE WARPED DGP BRANE

We start by writing down the Friedmann equation on the warped DGP brane, by using the Friedmann-Robertson-Walker (FRW) metric. It becomes

$$H^2 + \frac{k}{a^2} = \frac{1}{3\mu^2}\Big[\rho + \rho_0\big(1 + \epsilon\mathcal{A}(\rho, a)\big)\Big], \tag{1}$$

where $H$ is the Hubble parameter, $a$ represents the scale factor, $\rho$ is the total energy density, and $k$ is the constant curvature of the three-space of the FRW metric. The $\mu$ parameters denotes the strength of the induced gravity term on the brane. Also, the $\epsilon$ parameter becomes either $+1$ or $-1$ representing the two branches of this model. For $\epsilon = -1$ the brane tension can be assume to be positive, while for $\epsilon = +1$ this is negative. $\mathcal{A}$ is defined by

$$\mathcal{A} = \left[\mathcal{A}_0^2 + \frac{2\eta}{\rho_0}\left(\rho - \mu^2\frac{\mathcal{E}_0}{a^4}\right)\right]^{\frac{1}{2}}, \tag{2}$$

where

$$\mathcal{A}_0 = \sqrt{1 - 2\eta\frac{\mu^2\Lambda}{\rho_0}}, \quad \rho_0 = m_\lambda^4 + 6\frac{m_5^6}{\mu^2}, \quad \eta = \frac{6m_5^6}{\rho_0\mu^2} \quad (0 < \eta \leq 1), \tag{3}$$

and $\Lambda$ is defined by

$$\Lambda = \frac{1}{2}({}^{(5)}\Lambda + \frac{1}{6}\kappa_5^4\lambda^2), \tag{4}$$

where $\kappa_5$ is the 5-dimensional Newton constant, ${}^{(5)}\Lambda$ is the 5-dimensional cosmological constant in the bulk, and $\lambda$ is the brane tension. Note that there are three mass scales, $\mu$,

$m_\lambda = \lambda^{1/4}$ and $m_5 = \kappa_5^{-2/3}$. Also, the quantities $\mathcal{E}_0$ is a constant related to Weyl radiation. Since we are interested in the inflationary dynamics of the model, we will neglect the curvature term and the dark radiation term. We shall restrict ourselves to the RS critical case, i.e. $\Lambda = 0$. Then Eq.(1) becomes

$$H^2 = \frac{1}{3\mu^2}\left[\rho + \rho_0 + \epsilon\rho_0(1 + \frac{2\eta\rho}{\rho_0})^{1/2}\right]. \quad (5)$$

Note that in the ultra high energy limit where $\rho \gg \rho_0 \gg m_\lambda^4$, Eq.(5) results to be

$$H^2 = \frac{1}{3\mu^2}\left(\rho + \epsilon\sqrt{2\rho\rho_0}\right). \quad (6)$$

In the intermediate energy region where $\rho \ll \rho_0$ but $\rho \gg m_\lambda^4$, for the branch with $\epsilon = -1$, the Friedamnn equation reads $H^2 = \frac{m_\lambda^4}{18 m_5^6}\left(\rho + \frac{\rho^2}{2m_\lambda^4} - \frac{\mu^2 m_\lambda^4}{6m_5^6}\rho - \frac{\mu^2}{4m_5^6}\rho^2\right)$. Finally, when $\rho \ll m_\lambda^4 \ll \rho_0$ i.e. in low energy limit, Eq.(5) becomes

$$H^2 = \frac{1}{3\mu_p^2}\left[\rho + \mathcal{O}\left(\frac{\rho}{\rho_0}\right)^2\right], \quad (7)$$

where $\mu_p$ is the effective 4-dimensional Planck mass and is given by $\mu_p^2 = \mu^2/(1-\eta)$.

In the following we will consider a total energy density $\rho = \rho_\phi + \rho_\gamma$ where $\phi$ corresponds to a self-interacting scalar field with energy density, $\rho_\phi$, given by, $\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)$ and $\rho_\gamma$ represents the radiation energy density.

## III. WARM-DGP INFLATIONARY PHASE

The dynamics of the cosmological model in the warm-DGP inflationary scenario is described by the equations

$$\ddot{\phi} + 3H\,\dot{\phi} + V_{,\phi} = -\Gamma\;\dot{\phi}, \quad (8)$$

and

$$\dot{\rho}_\gamma + 4H\rho_\gamma = \Gamma\dot{\phi}^2. \quad (9)$$

Here $\Gamma$ is the dissipation coefficient and it is responsible of the decay of the scalar field into radiation during the inflationary era. $\Gamma$ can be assumed to be a constant or a function of the scalar field $\phi$ or the temperature $T$ or both [1]. Here, we will take $\Gamma$ to be a function of $\phi$ only. At the end subsection B of section V, we briefly describe the situation in which $\Gamma =$ const. In the near future we hope to study more realistic models in which $\Gamma$ not only

depends on $\phi$ but also on $T$, expression which could be derived from first principles via Quantum Field Theory approach [13, 14]. On the other hand, $\Gamma$ must satisfies $\Gamma = f(\phi) > 0$ by the Second Law of Thermodynamics. Dots mean derivatives with respect to time and $V_{,\phi} = \partial V(\phi)/\partial\phi$.

During the inflationary epoch the energy density associated to the scalar field is of the order of the potential, i.e. $\rho_\phi \sim V$, and dominates over the energy density associated to the radiation field, i.e. $\rho_\phi > \rho_\gamma$. Assuming the set of slow-roll conditions, i.e. $\dot{\phi}^2 \ll V(\phi)$, and $\ddot{\phi} \ll (3H + \Gamma)\dot{\phi}$ [1], the Friedmann equation (5) reduces to

$$H^2 \approx \frac{1}{3\mu^2}\left[V + \rho_0 + \epsilon\rho_0(\mathcal{A}_0^2 + \frac{2\eta\, V}{\rho_0})^{1/2}\right], \tag{10}$$

and Eq. (8) becomes

$$3H\,[\,1 + r\,]\,\dot{\phi} \approx -V_{,\phi}, \tag{11}$$

where $r$ is the rate defined as

$$r = \frac{\Gamma}{3H}. \tag{12}$$

For the high (weak) dissipation regimen, we have $r \gg 1$ ($r < 1$).

We also consider that during warm inflation the radiation production is quasi-stable, i.e. $\dot{\rho}_\gamma \ll 4H\rho_\gamma$ and $\dot{\rho}_\gamma \ll \Gamma\dot{\phi}^2$. From Eq.(9) we obtained that the energy density of the radiation field becomes

$$\rho_\gamma = \frac{\Gamma\dot{\phi}^2}{4H}, \tag{13}$$

which could be written as $\rho_\gamma = \sigma T_r^4$, where $\sigma$ is the Stefan-Boltzmann constant and $T_r$ is the temperature of the thermal bath. By using Eqs.(11), (12) and (13) we get

$$\rho_\gamma = \sigma\, T_r^4 = \frac{r\,\mu^2}{4\,(1+r)^2}\left[\frac{V_{,\phi}^2}{\left[V + \rho_0 + \epsilon\rho_0(\mathcal{A}_0^2 + \frac{2\eta\, V}{\rho_0})^{1/2}\right]}\right]. \tag{14}$$

Introducing the dimensionless slow-roll parameter we get

$$\varepsilon \equiv -\frac{\dot{H}}{H^2} = \frac{\mu^2}{2}\frac{1}{(1+r)}\left[\frac{V_{,\phi}}{V}\right]^2\left[\frac{1 + \epsilon\eta(\mathcal{A}_0^2 + 2\eta\, V/\rho_0)^{-1/2}}{\left[1 + \frac{\rho_0}{V}(1 + \epsilon(\mathcal{A}_0^2 + 2\eta\frac{V}{\rho_0})^{1/2})\right]^2}\right], \tag{15}$$

and the second slow-roll parameter $\alpha$ becomes

$$\alpha \equiv -\frac{\ddot{H}}{H\dot{H}} \simeq \frac{V_{,\phi\phi}}{3\,(1+r)\,H^2}. \tag{16}$$

We see that for $r = 0$ (or $\Gamma = 0$), the parameters $\varepsilon$ and $\alpha$ given by Eqs.(15) and (16) respectively, are reduced to the typical expression for cool inflation in the DGP brane[10]. Note that the term in the bracket of Eq.(15) is the correction to the standard warm inflationary model. Also, this term loses the condition for inflation, i.e. $\varepsilon \ll 1$, and when $\epsilon = 1$ and tightens the condition when $\epsilon = -1$, in contrast to the RS model where the correction term always loses the inflationary condition.

It is possible to find a relation between the energy densities $\rho_\gamma$ and $\rho_\phi$ given by

$$\rho_\gamma = \frac{r}{2(1+r)}\varepsilon\left[\frac{\rho_\phi + \rho_0(1+\epsilon[\mathcal{A}_0^2 + 2\eta\frac{\rho_\phi}{\rho_0}]^{1/2})}{1+\epsilon\eta(\mathcal{A}_0^2 + 2\eta\ \rho_\phi/\rho_0)^{-1/2}}\right] \simeq \frac{r}{2(1+r)}\varepsilon\left[\frac{V + \rho_0(1+\epsilon[\mathcal{A}_0^2 + 2\eta\frac{V}{\rho_0}]^{1/2})}{1+\epsilon\eta(\mathcal{A}_0^2 + 2\eta\ V/\rho_0)^{-1/2}}\right].$$

Recall that during inflation the energy density of the scalar field becomes dominated by the potential energy, i.e. $\rho_\phi \sim V$.

The condition which the warm inflation epoch on a DGP Brane could take place can be summarized with the parameter $\varepsilon$ satisfying the inequality $\varepsilon < 1$. This condition is analogue to the requirement that $\ddot{a} > 0$. The condition given above is rewritten in terms of the densities by using $\rho_\gamma$, we get

$$\left[\frac{\rho_\phi + \rho_0(1+\epsilon[\mathcal{A}_0^2 + 2\eta\frac{\rho_\phi}{\rho_0}]^{1/2})}{1+\epsilon\eta(\mathcal{A}_0^2 + 2\eta\ \rho_\phi/\rho_0)^{-1/2}}\right] > \frac{2(1+r)}{r}\ \rho_\gamma. \tag{17}$$

Inflation ends when the universe heats up at a time when $\varepsilon \simeq 1$, which implies

$$\left[\frac{V_{,\phi}}{V}\right]^2 \left[\frac{1+\epsilon\eta(\mathcal{A}_0^2 + 2\eta\ V/\rho_0)^{-1/2}}{\left[1+\frac{\rho_0}{V}(1+\epsilon(\mathcal{A}_0^2 + 2\eta\frac{V}{\rho_0})^{1/2})\right]^2}\right] \simeq \frac{2}{\mu^2}(1+r). \tag{18}$$

The number of e-folds at the end of inflation is given by

$$N = -3\int_{\phi_*}^{\phi_f} \frac{H^2}{V_{,\phi}}(1+r)d\phi'. \tag{19}$$

In the following, the subscripts $*$ and $f$ are used to denote to the epoch when the cosmological scales exit the horizon and the end of inflation, respectively.

## IV. PERTURBATIONS

In this section we will study the scalar and tensor perturbations for our model. Note that in the case of scalar perturbations the scalar and the radiation fields are interacting.

Therefore, isocurvature (or entropy) perturbations are generated besides of the adiabatic ones. This occurs because warm inflation can be considered as an inflationary model with two basics fields [15, 16]. In this context dissipative effects can produce a variety of spectral, ranging between red and blue [2, 15], and thus producing the running blue to red spectral suggested by WMAP three-year data[12].

As argued in Ref.[10] for the DGP brane (see also [8, 9, 17]), the density perturbation could be written as $\delta_H = \frac{2}{5}\frac{H}{\dot{\phi}}\,\delta\phi$ [18].

From Eqs.(11) and (12), the latter equation becomes

$$\delta_H^2 = \frac{36}{25}\frac{H^4\,r^2}{V_{,\phi}^2}\,\delta\phi^2. \tag{20}$$

The scalar field presents fluctuations which are due to the interaction between the scalar and the radiation fields. In the case of high dissipation, the dissipation coefficient $\Gamma$ is much greater that the rate expansion $H$ , i.e. $r = \Gamma/3H \gg 1$ and following Taylor and Berera[19], we can write

$$(\delta\phi)^2 \simeq \frac{k_F\,T_r}{2\,\pi^2}, \tag{21}$$

where the wave-number $k_F$ is defined by $k_F = \sqrt{\Gamma H/V} = H\sqrt{3r} \geq H$, and corresponds to the freeze-out scale at which dissipation damps out to the thermally excited fluctuations. The freeze-out wave-number $k_F$ is defined at the point where the inequality $V_{,\phi\phi} < \Gamma H$, is satisfied [19].

From Eqs. (20) and (21) it follows that

$$\delta_H^2 \approx \frac{18\sqrt{3}}{25\,\pi^2}\left[\frac{H^5\,T_r\,r^{5/2}}{V_{,\phi}^2}\right]. \tag{22}$$

The scalar spectral index $n_s$ is given by $n_s - 1 = \frac{d\ln\,\delta_H^2}{d\ln k}$, where the interval in wave number is related to the number of e-folds by the relation $d\ln k(\phi) = -dN(\phi)$. From Eq.(22), we get

$$n_s \approx 1 - \left[5\widetilde{\varepsilon} - 2\widetilde{\alpha} + \frac{5\,r_{,\phi}\,V_{,\phi}}{6\,r^2\,H^2}\right] \approx 1 - \frac{5\,\widetilde{\varepsilon}}{2} + 2\widetilde{\alpha} - \frac{5\,V_{,\phi}\,\Gamma_{,\phi}}{2\,H\,\Gamma^2}, \tag{23}$$

where, the slow-roll parameters $\widetilde{\varepsilon}$ and $\widetilde{\alpha}$, (for $r \gg 1$) are given by

$$\widetilde{\varepsilon} \approx \frac{\mu^2}{2\,r}\left[\frac{V_{,\phi}}{V}\right]^2\left[\frac{1+\epsilon\eta(\mathcal{A}_0^2+2\eta\,V/\rho_0)^{-1/2}}{\left[1+\frac{\rho_0}{V}(1+\epsilon(\mathcal{A}_0^2+2\eta\frac{V}{\rho_0})^{1/2})\right]^2}\right], \quad \text{and} \quad \widetilde{\alpha} \approx \frac{V_{,\phi\phi}}{3\,r\,H^2}, \tag{24}$$

respectively.

One of the interesting features of the three-year data set from WMAP is that it hints at a significant running in the scalar spectral index $dn_s/d\ln k = \alpha_s$ [12]. Dissipative effects themselves can produce a rich variety of spectra ranging between red and blue [2, 15]. From Eq.(23) we obtain that the running of the scalar spectral index becomes

$$\alpha_s = \frac{15\, r\, V_{,\phi}}{\Gamma^2}\,\left[\widetilde{\varepsilon}_{,\phi} - \frac{2\,\widetilde{\alpha}_{,\phi}}{5} - \frac{(\ln r)_{,\phi}}{2}\,[\widetilde{\alpha} - 2\widetilde{\varepsilon}] - \frac{3\,V_{,\phi}}{2\,\Gamma^2}\,\left(r_{,\phi\phi} - 2\frac{r_{,\phi}^2}{r}\right)\right]. \qquad (25)$$

In models with only scalar fluctuations the marginalized value for the derivative of the spectral index is approximately $-0.05$ from WMAP-three year data only [12].

As it was mentioned in Ref.[20] the generation of tensor perturbations during inflation would produce stimulated emission in the thermal background of gravitational wave. This process changes the power spectrum of the tensor modes by an extra temperature dependently factor given by $\coth(k/2T)$. The corresponding spectrum becomes

$$A_g^2 = \frac{16\pi}{\mu^2}\left(\frac{H}{2\pi}\right)^2 \coth\left[\frac{k}{2T}\right] \simeq \frac{4}{3\pi\mu^4}\,\left[V + \rho_0 + \epsilon\rho_0(\mathcal{A}_0^2 + \frac{2\eta\, V}{\rho_0})^{1/2}\right]\coth\left[\frac{k}{2T}\right], \qquad (26)$$

where the spectral index $n_g$, results to be given by $n_g = \frac{d}{d\ln k}\,\ln\left[\frac{A_g^2}{\coth[k/2T]}\right] = -2\,\varepsilon$. Here, we have used that $A_g^2 \propto k^{n_g}\,\coth[k/2T]$ [20].

For $r \gg 1$ and from expressions (22) and (26) we may write the tensor-scalar ratio as

$$R(k) = \left(\frac{A_g^2}{P_{\mathcal{R}}})\right)\Bigg|_{k_*} \simeq \frac{8\pi}{9\sqrt{3}\mu^2}\,\left[\frac{V_{,\phi}^2}{T_r\, H^3\, r^{5/2}}\,\coth\left(\frac{k}{2T}\right)\right]\Bigg|_{k=k_*}. \qquad (27)$$

Here, $\delta_H \equiv 2\,P_{\mathcal{R}}^{1/2}/5$ and $k_*$ is referred to $k = Ha$, the value when the universe scale crosses the Hubble horizon during inflation.

Combining WMAP three-year data[12] with the Sloan Digital Sky Survey (SDSS) large scale structure surveys [21], it is found an upper bound for $R$ given by $R(k_* \simeq 0.002$ Mpc$^{-1}$)$<$ 0.28 (95%$CL$), where $k_* \simeq$0.002 Mpc$^{-1}$ corresponds to $l = \tau_0 k \simeq 30$, with the distance to the decoupling surface $\tau_0$= 14,400 Mpc. The SDSS measures galaxy distributions at red-shifts $a \sim 0.1$ and probes $k$ in the range 0.016 $h$ Mpc$^{-1}$$< k <$0.011 $h$ Mpc$^{-1}$. The recent WMAP three-year results give the values for the scalar curvature spectrum $P_{\mathcal{R}}(k_*) \equiv 25\delta_H^2(k_*)/4 \simeq 2.3 \times 10^{-9}$ and the scalar-tensor ratio $R(k_*) = 0.095$. We will make use of these values to set constrains on the parameters for our model.

## V. EXPONENTIAL POTENTIAL IN THE HIGH DISSIPATION APPROACH

Let us consider an inflaton scalar field $\phi$ on the brane with exponential potential. This potential occur naturally in some fundamental theories such as string/M theories and it is intensively studied in DGP models.

We write for the exponential potential as $V = V_0 e^{-\sqrt{2/p}\,\frac{\phi}{\mu}}$, where $V_0$ and $p$ are two constants, and $V(\phi) \longrightarrow 0$ as $\phi \longrightarrow \infty$, which means $p > 0$. An estimation of these parameters are give for cool inflation on a DGP brane in Ref.[10]. In the following, we will restrict ourselves to the high dissipation regimen i.e. $r \gg 1$.

### A. $\Gamma \propto \left[v(\phi) + 1 + \epsilon(\mathcal{A}_0^2 + 2\eta\, v(\phi))^{1/2})\right]^{1/2}$ case.

This choice is motivated by the fact that $r$ becomes a constant. By using the exponential potential, we find that the slow-roll parameters become

$$\widetilde{\varepsilon} = \frac{1}{p\, r_0}\;\left[\frac{1 + \epsilon\eta(\mathcal{A}_0^2 + 2\eta\; v)^{-1/2}}{\left[1 + \frac{1}{v}(1 + \epsilon(\mathcal{A}_0^2 + 2\eta\, v)^{1/2})\right]^2}\right], \text{and}\; \widetilde{\alpha} = \frac{2}{p\, r_0}\;\left[\frac{v}{\left[v + 1 + \epsilon(\mathcal{A}_0^2 + 2\eta\, v)^{1/2})\right]^2}\right],$$

where we have define $v = \frac{V}{\rho_0}$.

By integrating Eq.(19) the number of e-folds results in $N = \frac{p\, r_0}{2}\,[h(v_f) - h(v_*)]$, where

$$h(v) = -\left[\ln v - v^{-1} + \left(W v^{-1} + \frac{2\eta}{\mathcal{A}_0}\tanh^{-1}(W/\mathcal{A}_0)\right)\right], \tag{28}$$

and $W = W(v) = (\mathcal{A}_0^2 + 2\eta\, v)^{1/2}$.

In section III we specified that the inflationary phase can exit naturally without any other mechanism in the branch $\epsilon = -1$. In this case the condition for inflation end, $\varepsilon = 1$, gives a quintuple equation that can not be solved analytically. Note that the same happens in cool inflation for the DGP brane [10].

From Eq.(22), when $r \gg 1$, we obtain that the scalar power spectrum becomes

$$P_{\mathcal{R}}(k) \approx \;\frac{p\; r_0^{5/2}}{4\,\pi^2}\;\left[\frac{\rho_0^{1/2}\, T_r\; v^{1/2}}{\mu^3}\right]\;\left[1 + \frac{1}{v}\,(1 + \epsilon\; W)\right]^{5/2}\Bigg|_{k=k_*}, \tag{29}$$

and from Eq.(27) the tensor-scalar ratio, is given by

$$R(k) \approx \;\frac{16\,\pi}{3}\left[\frac{\rho_0^{1/2}\;\, v^{1/2}}{\mu\; p\; r_0^{5/2}\; T_r}\right]\;\left[1 + \frac{1}{v}\,(1 + \epsilon\; W)\right]^{-3/2}\;\coth\left[\frac{k}{2T}\right]\Bigg|_{k=k_*}. \tag{30}$$

By using the WMAP three year data where $P_{\mathcal{R}}(k_*) \simeq 2.3 \times 10^{-9}$, $R(k_*) = 0.095$, and choosing the parameter $T \simeq T_r$, we obtained from Eqs.(29) and (30) that $v_* = 2\left[\frac{C-1+\eta}{(C-1)^2}\right]$, where the $C$ is given by $C \simeq \frac{5.1\times 10^{-10}}{v_\mu\ \coth\left[\frac{k_*}{2T_r}\right]}$, and $v_\mu = V_*/\mu^4$. Recall that $\Lambda = 0 \Longrightarrow \mathcal{A}_0 = 1$ and $\epsilon = -1$.

Now we consider the special case in which we fixe $N = 60$, $p = 5$ and $r_0 = 10$. In this special case we obtained that $v_f = 0.05$, $v_* = 36$ and $v_\mu = 7.5 \times 10^{-13}$. Also, we choose the parameters $k_* = 0.002$ Mpc$^{-1}$ and $T \simeq T_r \simeq 0.24 \times 10^{16}$ GeV. We also take $\eta = 0.99$, although the model can be used for any value of $\eta$.

From Eqs.(3) and (4) we obtained that the tension of the brane, $\lambda$ becomes $\frac{\lambda}{\mu^4} = \frac{\rho_0}{\mu^4}\,(1-\eta) \simeq\ 2.1\times 10^{-16}$, and the 5-dimensional gravitational constant results to be $\frac{m_5}{\mu} = \left[\frac{\eta\,\rho_0}{6\,\mu^4}\right]^{1/6} \simeq 0.0039$. Analogously, we get that the cosmological constant ${}^5\Lambda$ in the bulk can be evaluate and it becomes ${}^5\Lambda/\mu^2 \simeq 2.1 \times 10^{-18}$. We note that the $\Lambda$, $m_5$ and ${}^5\Lambda$ parameters become decrease by two, one and fourth orders of magnitude, respectively, when are compared with those analogous cool article related to inflation in the DGP brane.

### B. $\Gamma \propto \left[v(\phi) + 1 + \epsilon(\mathcal{A}_0^2 + 2\eta\, v(\phi))^{1/2})\right]^{3/2}$ case.

This choice allows us to get $r$ as a function of $H$ in analogy with that one used by authors of Refs.[22, 23]. Therefore, we take $r$ to be of the form, $r = (3\,\mu^2/\rho_0)H^2$, and thus we find for the slow-roll parameters

$$\widetilde{\varepsilon} = \frac{1}{p\,v}\ \left[\frac{1+\epsilon\eta(\mathcal{A}_0^2+2\eta\ v)^{-1/2}}{\left[1+\frac{1}{v}(1+\epsilon(\mathcal{A}_0^2+2\eta\,v)^{1/2})\right]^3}\right],\ \text{and}\ \ \widetilde{\alpha} = \frac{2}{p\,v}\ \left[\frac{1}{\left[v+1+\epsilon(\mathcal{A}_0^2+2\eta\,v)^{1/2})\right]^2}\right].$$

The number of e-folds is given by $N = \frac{p}{2}\,[\Im(v_f) - \Im(v_*)]$, where the function $\Im(v)$ is defined by

$$\Im(v) = -\left[v - \left(\frac{1+\mathcal{A}_0^2\,\epsilon^2}{v}\right) + 2\,(1+\eta\,\epsilon)\ln v + \left(\epsilon\,W\,\left[4-\frac{2}{v}\right] + \frac{4\,\epsilon(\mathcal{A}_0^2+\eta)}{\mathcal{A}_0^2}\tanh^{-1}(W/\mathcal{A}_0)\right)\right].$$

The $N$ parameter has to have an appropriated values (60 or so) in order to solve the standard cosmological puzzles. In order to do so we need the following inequality $\Im(v_*) < \Im(v_f) - 120/p$ with $\Im(v_f) > 120/p$ to be satisfied. Again, the condition for inflation becomes to an end is $\varepsilon(v_f) = 1$, has roots that can not be written down in closed algebraic form.

At the epoch when the cosmological scale exit the horizon, the dissipation parameter becomes $r(\phi_*) = r_* \gg\ 1 \Longrightarrow\ \rho_0 \ll\ 3\,\mu^2\,H_*^2$, resulting in the requirement that $[v_* + 1 + \epsilon(\mathcal{A}_0^2 + 2\eta\,v_*)^{1/2}]\ \gg\ 1$.

From Eq.(22), and for $r \gg 1$, we obtain that the scalar power spectrum becomes

$$P_{\mathcal{R}}(k) \approx \frac{p}{4\,\pi^2}\left[\frac{\rho_0^{1/2}\,T_r\,v^3}{\mu^3}\right]\,\left[1+\frac{1}{v}\,(1+\epsilon\,W)\right]^5\Bigg|_{k=k_*}, \tag{31}$$

and from Eq.(27) the tensor-scalar ratio, is given by

$$R(k) \approx \frac{24\,\pi}{p}\left[\frac{\rho_0^{1/2}}{\mu\,T_r\,v^2}\right]\,\left[1+\frac{1}{v}\,(1+\epsilon\,W)\right]^{-4}\coth\left[\frac{k}{2T}\right]\Bigg|_{k=k_*}. \tag{32}$$

By using the WMAP three year data, where $P_{\mathcal{R}}(k_*) \simeq 2.3\times 10^{-9}$, $R(k_*) = 0.095$, and choosing the parameter $T \simeq T_r$, we obtained from Eqs.(31) and (32) that $v_* = 2\left[\frac{C-1+\eta}{(C-1)^2}\right]$, where now $C$ is given by $C \simeq \frac{1.1\times 10^{-10}}{v_\mu\,\coth\left[\frac{k_*}{2T_r}\right]}$, and $v_\mu = V_*/\mu^4$. Here, once again we have taken $\Lambda = 0 \Longrightarrow \mathcal{A}_0 = 1$ and $\epsilon = -1$.

Let us consider some numerical situations. For $N = 60$ and $p = 60$, we obtain that $v_f = 0.63$, $v_* = 8$ and $v_\mu = 2.1\times 10^{-13}$, where again we have taken the parameters $k_* = 0.002$ Mpc$^{-1}$, $T \simeq T_r \simeq 0.24\times 10^{16}$ GeV and $\eta = 0.99$.

From Eqs.(3) and (4), the tension of the brane becomes $\frac{\lambda}{\mu^4} = \frac{\rho_0}{\mu^4}\,(1-\eta) \simeq 2.6\times 10^{-16}$, and the 5-dimensional gravitational constant can be written as $\frac{m_5}{\mu} = \left[\frac{\eta\,\rho_0}{6\,\mu^4}\right]^{1/6} \simeq 0.0041$. Analogously, we obtain that ${}^5\Lambda/\mu^2 \simeq 2.7\times 10^{-18}$. Note that theses values are similar to their analogous for the case in which $r = r_0$.

In Fig.(1) we plot the scalar spectrum index $n_s$ and the running spectral index $\alpha_s$ versus the ratio $v = V/\rho_0$. In doing this, we have taken three different values of the parameter p. Note that in considering the $n_s$ versus $v = V/\rho_0$ plot the WMAP-three data favors hight values of this parameter. However, from the plot $\alpha_s$ versus $v = V/\rho_0$ we could not say the same, since it seems that any value of p would give the same value of $\alpha_s$ for hight enough $v = V/\rho_0$.

Another interesting case to study is to consider $\Gamma = \Gamma_0 = constant$ i.e. $r \propto H^{-1}$. Analogously as in the other cases, we take the special case with fixed $N = 60$ and $p = 80$. For these values, we have $v_f = 0.05$, $v_* = 50$, $v_\mu = 1.5\times 10^{-13}$ and $\Gamma_0 = 8\times 10^{15}$ GeV. Here, again we have taken the parameters $k_* = 0.002$ Mpc$^{-1}$, $T \simeq T_r \simeq 0.24\times 10^{16}$ GeV and $\eta = 0.99$. From Eqs.(3) and (4), the tension result to be $\frac{\lambda}{\mu^4} \simeq 2.9\times 10^{-17}$, and the 5-dimensional gravitational constant becomes $\frac{m_5}{\mu} \simeq 0.0028$. Finally, we obtain that ${}^5\Lambda/\mu^2 \simeq 3.0\times 10^{-19}$. Note that these values are similar to those values in which $\Gamma$ is variable.

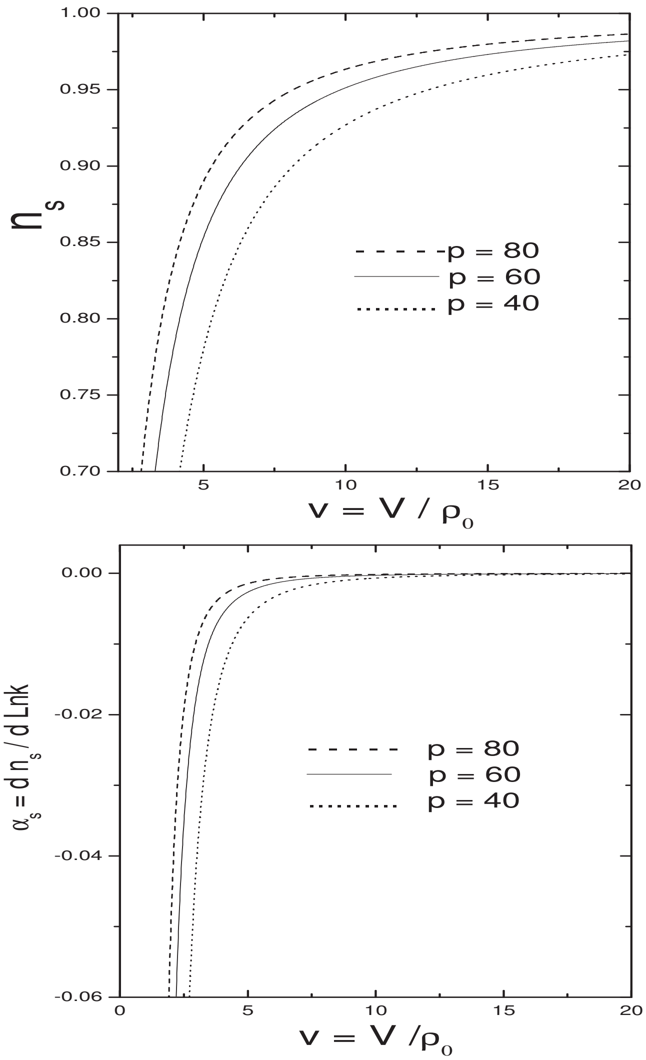


FIG. 1: The upper panel show the evolution of the scalar spectrum index $n_s$ versus $v = V/\rho_0$, and the lower panel show the evolution of the running scalar spectrum index, $\alpha_s$, versus $v = V/\rho_0$. In both panels we have used different values for the parameter $p$, for $r \propto H^2$. Here, we have taken the values $\eta = 0.99$, $\mathcal{A}_0 = 1$, and $\epsilon = -1$.

## VI. CONCLUSIONS

In this paper we have investigated the warm inflationary scenario on a warped DGP brane. In the slow-roll approximation we have found a general relationship between the radiation and scalar field energy densities. This has led us to a general criterium for warm inflation in DGP brane to occur (see Eq.(17)).

Our specific models are described by an exponential potential and we have consider different cases for the dissipation coefficient, $\Gamma$. In the first case, we took $\Gamma \propto$

$\left[v(\phi)+1+\epsilon(\mathcal{A}_0^2+2\eta\, v(\phi))^{1/2})\right]^{1/2}$. Here, we have found that the condition for inflation to end presents the same characteristic that occurs in cool inflation for the DGP brane [10], except that it depends on the extra parameter $r_0$. For the case in which the dissipation coefficient $\Gamma$ is taken to be a function of the scalar field, i.e. $\Gamma \propto \left[v(\phi)+1+\epsilon(\mathcal{A}_0^2+2\eta\, v(\phi))^{1/2})\right]^{3/2}$, it was possible to describe an appropriate warm inflationary universe model on DGP brane. In these cases, we have obtained the explicit expressions for the corresponding scalar spectrum index and the running of the scalar spectrum index. We have also study the situation in which $\Gamma = \Gamma_0 = Const$. Here, we have found that the values are similar to those found when $\Gamma$ is a function of the scalar field.

By using the WMAP three year data and consider a special case with fixed $N$, $p$ and $r_0$, we have found the values of the parameters $\lambda$, ${}^5\Lambda$ and $m_5$. Using the above parameters we will check with a numerical example that the evolution of the universe really undergoes a 4-dimensional stage, then a 5-dimensional stage, and finally a 4-dimensional stage again. From Eqs.(6) to (7) the dimensional transition occurs if $\lambda\mu^2/6m_5^6 \ll 1$ [7]. In our models that we have worked out we have obtained that $\lambda/\mu^4 \simeq 2\times 10^{-16}$, $m_5/\mu \simeq 0.004$ and therefore $\lambda\mu^2/6m_5^6 = 0.008 \ll 1$. Also, $v_* \sim \rho_*/\rho_0 \sim 10$ for these models and thus the universe inflates in a 4-dimensional stage when the cosmic scale crossed the Hubble horizon during inflation, since $\rho_* \gg \rho_0 \gg m_\lambda^4$. In the intermediate energy region (5-dimensional), it is necessary that $m_\lambda^4 \ll \rho \ll \rho_0$ and using the final value of $v_f \sim \rho_f/\rho_0$, we find that $\rho_f \ll 1$ and $\rho_f/m_\lambda^4 \sim 10$. Therefore, the inflationary phase exists in a 5-dimensional stage. Finally, the universe stops inflating and enters in a radiation dominated Big-Bang phase, in which, the energy density decreases very fast and the universe becomes 4-dimensional again.

We should note that other properties of this model deserves further study. For example, we did not study bispectrum of density perturbations. While cool inflation typically predicts a nearly vanishing bispectrum, and hence a small (just a few per cent) deviation from Gaussianity in density fluctuations -see e.g. [24], warm inflation clearly predicts a non-vanishing bispectrum. We left this question for a subsequent study of the warm-DGP brane-world. Also, a more accurate calculation for the density perturbation would be necessary in order to check the validity of expression (20). We intend to return to this point in the near future by working an approach analogous to that followed in Refs.[22] and [23].

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